AP Calculus BC – McGlone
Section 10.6 – Tangents (Anton new)

Find the slope of the tangent line to the polar curve for the given value of θ.

1. $r = 2\sin\theta\,;\ \theta = \frac{\pi}{6}$

2. $r = 1 + \cos\theta\,;\ \theta = \frac{\pi}{2}$

3. $r = \frac{1}{\theta};\ \theta = 2$

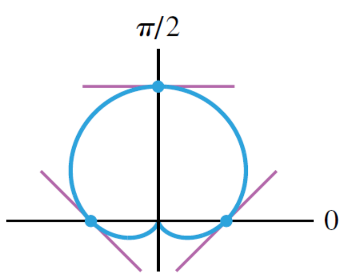
4. $r = a\sec\theta\,;\ \theta = \frac{\pi}{6}$

5. $r = \sin 3\theta\,;\ \theta = \frac{\pi}{4}$

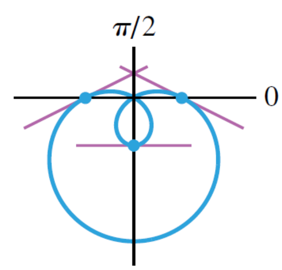
6. $r = 4 - 3\sin\theta\,;\ \theta = \pi$

Calculate the slopes of the tangent lines indicated in the accompanying figures.

7. $r = 2 + 2\sin\theta$



8. $r = 1 - 2\sin\theta$



Sketch the polar curve and find polar equations of the tangent lines to the curve at the pole.

9. $r = 2\cos 3\theta$

10. $r = 4\sin\theta$

11. $r = 4\sqrt{\cos 2\theta}$

12. $r = \sin 2\theta$

13. $r = 1 - 2\cos\theta$

14. $r = 2\theta$

Use a graphing utility to make a conjecture about the number of points on the polar curve at which there is a horizontal tangent line. Confirm your conjecture by finding the angles at which these tangents occur.

15. $r = \sin\theta\cos^2\theta$

16. $r = 1 - 2\sin\theta$

AP Calculus BC – McGlone
Section 10.6 – Tangents (Anton new)

Find the slope of the tangent line to the polar curve for the given value of θ.

1. $r = 2\sin\theta\,;\ \theta = \frac{\pi}{6}$

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{6}} = \left.\frac{\frac{d}{d\theta}\left(\cancel{2}\sin^2\theta\right)}{\frac{d}{d\theta}\left(\cancel{2}\sin\theta\cdot\cos\theta\right)}\right| = \left.\frac{2\sin\theta\cos\theta}{\sin\theta\cdot\sin\theta+\cos\theta\cdot\cos\theta}\right|_{\theta=\pi/6} = \frac{\cancel{2}\cdot\frac{1}{\cancel{2}}\cdot\frac{\sqrt{3}}{2}}{\frac{1}{2}\cdot\frac{-1}{2}+\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{3}}{2}} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

2. $r = 1+\cos\theta\,;\ \theta = \frac{\pi}{2}$

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{2}} = \left.\frac{\frac{d}{d\theta}\left((1+\cos\theta)\sin\theta\right)}{\frac{d}{d\theta}\left((1+\cos\theta)\cos\theta\right)}\right|_{\theta=\pi/2} = \left.\frac{(1+\cos\theta)(\cos\theta)+\sin\theta(-\sin\theta)}{(1+\cos\theta)(-\sin\theta)+\cos\theta(-\sin\theta)}\right|_{\theta=\pi/2} = \frac{-1}{-1} = 1$$

3. $r = \frac{1}{\theta};\ \theta = 2$

$$\left.\frac{dy}{dx}\right|_{\theta=2} = \left.\frac{\frac{d}{d\theta}\left(\frac{1}{\theta}\cdot\sin\theta\right)}{\frac{d}{d\theta}\left(\frac{1}{\theta}\cdot\cos\theta\right)}\right|_{\theta=2} = \left.\frac{\frac{1}{\theta}\cdot\cos\theta+\sin\theta\cdot\frac{-1}{\theta^2}}{\frac{1}{\theta}\cdot-\sin\theta+\cos\theta\cdot\frac{-1}{\theta^2}}\right|_{\theta=2} = \frac{\frac{1}{2}\cos 2-\frac{1}{4}\sin 2}{-\frac{1}{2}\sin 2-\frac{1}{4}\cos 2}$$

4. $r = a\sec\theta\,;\ \theta = \frac{\pi}{6}$

$r\cos\theta = a \Rightarrow x = a$ VERTICAL LINE $\Rightarrow \left.\frac{dy}{dx}\right|_{\theta=\pi/6}$ IS UNDEFINED.

5. $r = \sin 3\theta\,;\ \theta = \frac{\pi}{4}$

$$\left.\frac{dy}{dx}\right|_{\theta=\pi/4} = \left.\frac{\frac{d}{d\theta}\left(\sin 3\theta\cdot\sin\theta\right)}{\frac{d}{d\theta}\left(\sin 3\theta\cos\theta\right)}\right|_{\theta=\pi/4} = \left.\frac{\sin 3\theta\cdot\cos\theta+\sin\theta\cdot 3\cos 3\theta}{\sin 3\theta\cdot-\sin\theta+\cos\theta\cdot 3\cos 3\theta}\right|_{\theta=\pi/4} = \frac{\overset{-4}{\frac{\sqrt{2}}{\cancel{2}}\cdot\frac{\sqrt{2}}{\cancel{2}}+\frac{\sqrt{2}}{\cancel{2}}\cdot 3\cdot\frac{-\sqrt{2}}{\cancel{2}}}}{\underset{-8}{-\frac{\sqrt{2}}{\cancel{2}}\cdot\frac{\sqrt{2}}{\cancel{2}}+3\frac{\sqrt{2}}{\cancel{2}}\cdot\frac{-\sqrt{2}}{\cancel{2}}}}$$

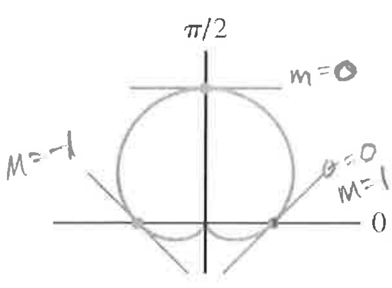
$= 1/2$

6. $r = 4-3\sin\theta\,;\ \theta = \pi$

$$\left.\frac{dy}{dx}\right|_{\theta=\pi} = \left.\frac{\frac{d}{d\theta}\left((4-3\sin\theta)\sin\theta\right)}{\frac{d}{d\theta}\left((4-3\sin\theta)\cos\theta\right)}\right|_{\theta=\pi} = \left.\frac{(4-3\sin\theta)\cos\theta+\sin\theta(-3\cos\theta)}{(4+3\sin\theta)(-\sin\theta)+\cos\theta(-3\cos\theta)}\right|_{\theta=\pi} = \frac{4(-1)+0}{(4)(0)-3(1)} = 4/3$$

Calculate the slopes of the tangent lines indicated in the accompanying figures.

7. $r = 2+2\sin\theta$

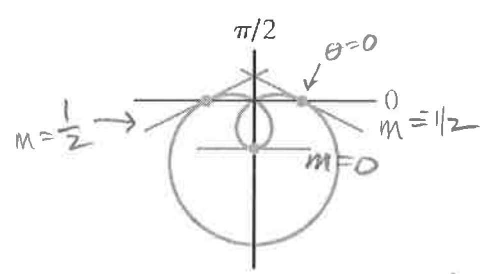


$$\left.\frac{dy}{dx}\right|_{\theta=0} = \left.\frac{\frac{d}{d\theta}\left((2+2\sin\theta)\sin\theta\right)}{\frac{d}{d\theta}\left((2+2\sin\theta)\cos\theta\right)}\right|_{\theta=0}$$

$$= \left.\frac{(2+2\sin\theta)(\cos\theta)+\sin\theta(2\cos\theta)}{(2+2\sin\theta)(-\sin\theta)+\cos\theta(2\cos\theta)}\right|_{\theta=0}$$

$$= \frac{(2)(1)+0}{(2)(0)+2} = 1$$

8. $r = 1-2\sin\theta$



$$\left.\frac{dy}{dx}\right|_{\theta=0} = \left.\frac{\frac{d}{d\theta}\left((1-2\sin\theta)\sin\theta\right)}{\frac{d}{d\theta}\left((1-2\sin\theta)\cos\theta\right)}\right|_{\theta=0}$$

$$= \left.\frac{(1-2\sin\theta)\cos\theta+\sin\theta(-2\cos\theta)}{(1-2\sin\theta)(-\sin\theta)+\cos\theta(-2\cos\theta)}\right|_{\theta=0}$$

$$= \frac{(1)(1)+0}{(1)(0)+2} = 1/2$$

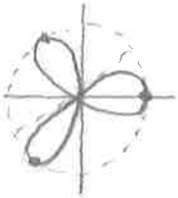
Sketch the polar curve and find ~~polar~~ equations of the tangent lines to the curve at the pole.

9. $r = 2\cos 3\theta = 0$

$\cos 3\theta = 0$

$3\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$

$\theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6},$



$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{6}} = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$

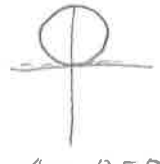
$\Rightarrow \boxed{y = \frac{1}{\sqrt{3}}x}$

$\left.\frac{dy}{dx}\right|_{\theta=\pi/2} = \tan\frac{\pi}{2} = \phi$

$\boxed{x = 0}$

$\left.\frac{dy}{dx}\right|_{\theta=\frac{5\pi}{6}} = -\frac{1}{\sqrt{3}} \Rightarrow \boxed{y = \frac{-1}{\sqrt{3}}x}$

10. $r = 4\sin\theta$ $\boxed{y = 0}$



$\left.\frac{dy}{dx}\right|_{\theta=0} = \tan 0 = 0$

$4\sin\theta = 0$

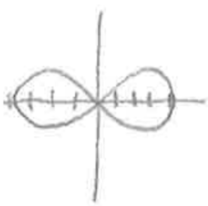
$\theta = 0$

11. $r = 4\sqrt{\cos 2\theta} = 0 \Rightarrow \cos 2\theta = 0$

$2\theta = \pi/2, 3\pi/2,$

$\theta = \pi/4, 3\pi/4$

$r^2 = 16\cos 2\theta$



$\left.\frac{dy}{dx}\right|_{\theta=\pi/4} = \tan\pi/4 = 1 \Rightarrow \boxed{y = x}$

$\left.\frac{dy}{dx}\right|_{\theta=\frac{3\pi}{4}} = \tan\frac{3\pi}{4} = -1 \Rightarrow \boxed{y = -x}$

12. $r = \sin 2\theta = 0 \Rightarrow 2\theta = 0, \pi, 2\pi, \ldots$

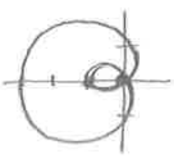
$\theta = 0, \pi/2, \pi, 3\pi/2.$

$\left.\frac{dy}{dx}\right|_{\theta=0,\pi,\ldots} = \tan 0 = 0 \Rightarrow \boxed{y = 0}$

$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{2},\frac{3\pi}{2}} = \tan\frac{\pi}{2} = \phi \Rightarrow \boxed{x = 0}$

13. $r = 1 - 2\cos\theta = 0 \Rightarrow \cos\theta = 1/2$

$\theta = \pi/3, 5\pi/3$



$\left.\frac{dy}{dx}\right|_{\theta=\pi/3} = \tan\pi/3 = \sqrt{3} \Rightarrow \boxed{y = \sqrt{3}x}$

$\left.\frac{dy}{dx}\right|_{\theta=\frac{5\pi}{3}} = \tan\frac{5\pi}{3} = -\sqrt{3} \Rightarrow \boxed{y = -\sqrt{3}x}$

14. $r = 2\theta = 0$ AT $\theta = 0$ $\tan 0 = 0$

$\Rightarrow \boxed{y = 0}$

Use a graphing utility to make a conjecture about the number of points on the polar curve at which there is a horizontal tangent line. Confirm your conjecture by finding the angles at which these tangents occur.

15. $r = \sin\theta\cos^2\theta$

$\frac{d}{d\theta}(\sin^2\theta\cos^2\theta) = 0$

$0 = 2\cos^3\theta\sin\theta - 2\cos\theta\sin^3\theta$

$0 = 2\cos\theta\sin\theta(\cos^2\theta - \sin^2\theta)$

$\theta = \underbrace{0, \pi}_{\text{SAME TANGENT}}, \underbrace{\cancel{\pi/2}}_{\text{VERT TANG.}}, \pi/4, 3\pi/4$

$\boxed{\theta = 0, \pi/4, 3\pi/4.}$

16. $r = 1 - 2\sin\theta$

$\frac{d}{d\theta}((1 - 2\sin\theta)\sin\theta) = 0$

$0 = \cos\theta - 4\sin\theta\cos\theta$

$0 = \cos\theta(1 - 4\sin\theta)$

$\cos\theta = 0$ $\sin\theta = 1/4$

$\theta = \pi/2, 3\pi/2$ $\theta = \underbrace{.253, 2.888}_{\text{SAME HORIZ. TANG.}}$